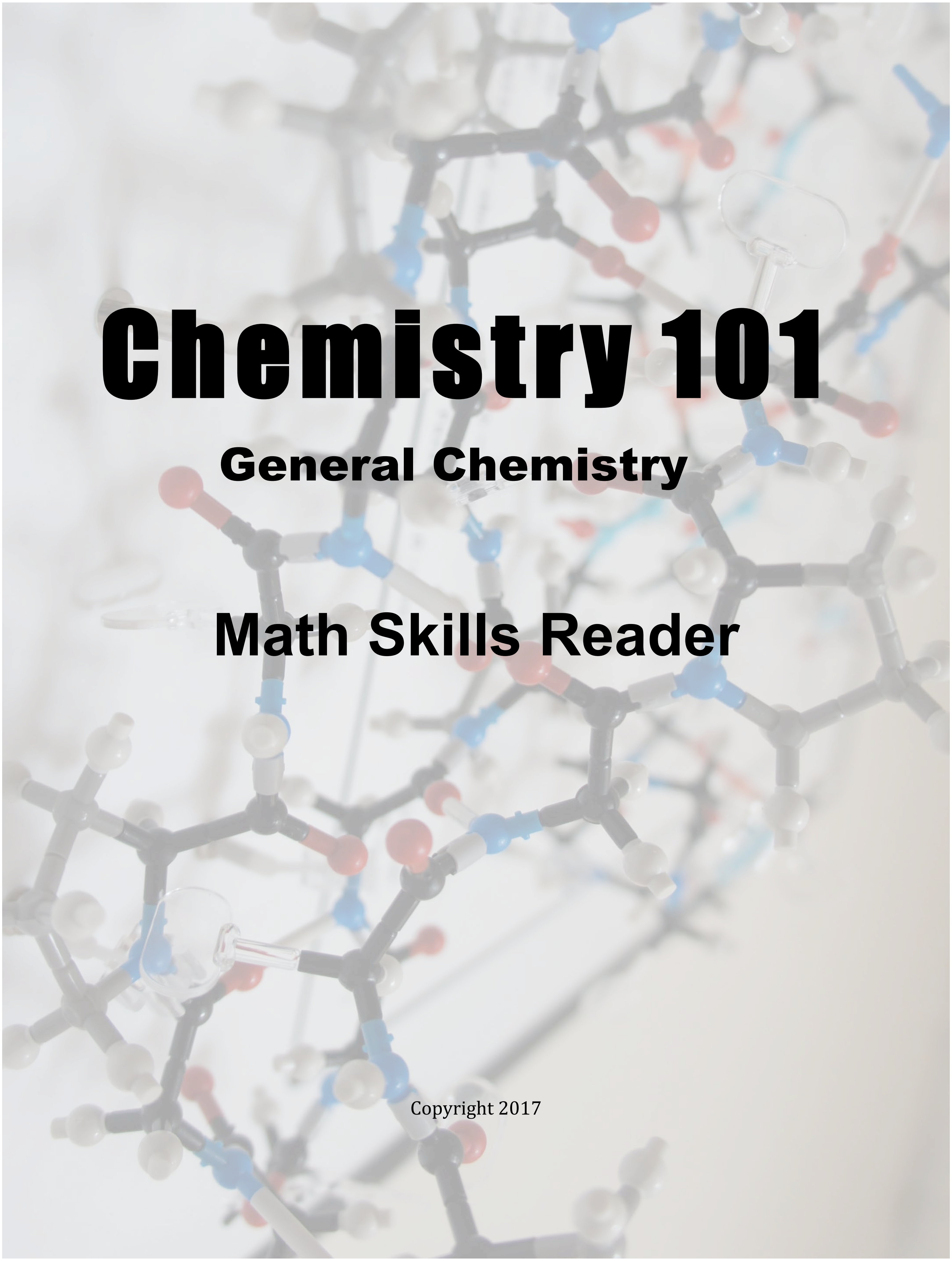

Chemistry 101

General Chemistry

Math Skills Reader

Part 1: Units, Significant Figures and Conversions

Numbers are part of math. You are very familiar with numbers – in fact in math there are two kinds of numbers – integers and real numbers. So we get stuff like 4 or 2.33 or even p. Real numbers can be rational or irrational. At the end of the day, though, these are numbers.

Chemistry (and science in general) does a lot more with numbers. Yes, sometimes we have numbers, but so many of our numbers also have units. Think about the following question:

Q: How far is it to the store?

A: About 3

Three what??? You would say. Is it 3 miles, 3 blocks? You instinctively wanted to hear a *unit* attached to the number.

In chemistry we are going to think about:

1) Pure numbers. These are actually rare although p comes up a lot. These numbers can be irrational, rational, or even integers. But the important thing about pure numbers is *they do not have units*.
2) Constants or universal constants. These are numbers with units. We will look at some of these in just a minute.
3) Variables that have *values* in specific circumstances. These variables have units. The actual value of the variable changes depending upon the circumstances.

Units

Units matter. They matter a lot. In fact, you use units all the times. Think about questions you might ask that require number answers with units?

How much does this cost?

How far will we go?

How long will it take?

You would never answer these questions with just a number. The units (dollars, miles, days) belong to specific categories. Let's think about some categories and put them in the table below

Unit category	
Length	
Time	
Volume	
Mass	

You can probably think of some more unit categories, but let's work with these. Now think of examples that can go in the unit category. For example, length could be miles, inches, meters, centimeters. Let's put these examples in the table – and let's use their common abbreviations.

Unit category	Examples
Length	*mi, m, cm, in, mm, ft, yds*
Time	*s, hrs, d, ns*
Volume	*l, fl oz, cm^3, dl, gal, qt*
Mass	*g, kg, lbs (? We'll come back to this)*

Many of these example units are simple units. The volume units, when expressed in length, are length cubed. This is an example of a more complex unit. The units will get even more complex shortly.

It is also common to put units into *unit systems*. For example, there is the metric system and the British system. So let's update our table:

Unit category	Metric System	British System
Length	*m, cm, mm*	*mi, in, ft, yds*
Time	*s, hrs, d, ns*	*s, hrs, d, ns*
Volume	*l, dl*	*fl oz, gal, qt*
Mass	*g, kg*	*lbs*

Now if we were anywhere else in the world beside the US, we would be concerned with the British system. Even in the US, scientists only use the metric system. For this course we will almost always be working with the metric system – but you need to be aware of the relationship between the Metric and British systems.

As scientists and science students we are going to work only in the metric system. But as you can see in the table, there are still lots of unit choices. Scientists have gotten together (and continue to get together) to choose a set of units that all scientists will use. This helps scientists communicate their results – we don't have to be converting among each other's favorite unit systems.

The system of units used by all scientists is called the SI system (International System of Units – go look it up to see its history). There are seven base SI units -- all other units in science can be derived from these seven. Go check out this link:
http://physics.nist.gov/cuu/Units/SIdiagram.html

Unit category	SI units
Length	*Meter (m)*
Time	*Second (s)*
Mass	*Kilogram (kg)*
Temperature	*Kelvin (K)*
Current	*Amp (A)*
Luminosity	*Candela (cd)*
Amount of substance	*Mole (mol)*

In chemistry we use length, time, mass, temperature, and amount of substance all of the time. Sometimes we use current and only rarely do we use luminosity.

As should be apparent now our numbers in science have meaning. They have meaning in that they are describing something (a distance) but the actual number itself also means something. The following numbers are actually different:

5 5.0 5.00 5.000

They are different because they are telling us something about the *error* inherent in the number. Most numbers in science are actually *measured* and therefore there is error inherent in the measurement. Even if number are carefully, carefully measured, there is still error. There is a limit to the accuracy of every measurement.

Consider measuring mass on a balance that can only measure to the tenth of a gram. You measure your substance to be *6.2 grams*. Now is it really 6.18523 grams or is it 6.20259 grams? You don't know. You can't know. That means the numbers you are reporting has *significant* figures. You in fact want to report the figures that are *significant* and leave out those that are irrelevant.

For example, suppose you measure the mass of a metal ball bearing to be *4.4 g.* You then decide to measure the volume of the ball bearing. You use a graduated cylinder (go look up what one looks like if you have never seen one) to measure volume and the cylinder can only measure volume to the tenths of a milliliter. So you measure the volume of the ball bearing to be *3.4 ml*. Now we might want the density of the ball bearing. The density is mass/volume. I go to be my calculator and divide 4.4 by 3.4 and get 1.294117647! OK, that's the accurate number that results from the division. *But remember in science numbers convey lots of information – including something about the accuracy of measurement*. There is <u>no way</u> we could know the density to nine decimal places when we only measured the mass and the volume to one decimal place. Thus our density calculation produced a lot of figure but they are most irrelevant – the actual answer should have only 2 figures!

How do I know this? We know turn to the logical, but tedious, task of learning or reviewing significant figures. First, I wanted to convince you that the significant figure world is really about *measurement*. That means you have to pay a lot of attention to the figure in the laboratory. But there are also rules that we can follow. If you think about it, these rules will actually make a lot of sense.

Here they are:

Operation	Rule	Example
Add, Subtract	Find the number that is being added or subtracted that has the *fewest* numbers to the right of the decimal point. The sum or difference should have the same number of decimal places.	4.23 + 6.891 = 11.12 4.23 + 6.899 = 11.13 (follow rules of rounding) 0.08 + 10.2 = 10.3
Multiply, Divide	Find the number that is being multiplied or divided that has the *fewest digits*. This time you don't just look at decimal places – just count digits. The answer should have the same number of digits.	6.2 x 1.232 = 7.6 38.45/26.1 = 1.47

There are a few complications to the above rules. These complications all involve how we think about zeroes. Before we can study the complications we need to review scientific notation.

Scientific notation

Again, scientists agree on a *convention*. We agree to represent all numbers in one way using exponential notation. Look at the following numbers and observe the trends:

Number	**Represented in scientific notation**
4.3	4.3
12.25	1.225×10^{1}
0.00823	8.32×10^{-3}

Numbers represented in scientific notation are always reported as a number between 1 and 10 (called the coefficient) multiplied by the appropriate base 10 power. Numbers with zeroes in it can be tricky. Zeroes can just be place holders – as are any zeroes to the left of the first non-zero number.

$0.000342 = 3.42 \times 10^{-4}$. This number has 3 significant figures

$0.0003042 = 3.042 \times 10^{-4}$. The zeroes to the left of the 3 are placeholders, the zero between the 3 and 4 is significant.

Think about this rule – convince yourself because the rules makes sense.

So now our multiplication (and division) rules can now include the zero rule. To see significant figures more clearly, put everything your are multiplying or dividing into scientific notation and then count the digits in the coefficient:

$0.008312 * 101.23 = 8.312 \times 10^{-3} \times 1.0123 \times 10^{2} = 8.414 \times 10^{-1}$

Note: It is important that you can use the scientific notation buttons of your calculator and that you know the rules of exponents. If you are hazy on this, now would be a good time to review. There are lots of good websites to help with the review.

Now I say the following not out of meanness but out of accuracy:

We expect you to be completely, totally, thoroughly proficient with this math. We consider it 8th grade mathematics. Now we also know some of you are hazy on this. *Go master it!!!* You should be able to do the following in your sleep:

Multiply (or divide) numbers with exponents

Use negative exponents (and know what they mean)

Add (or subtract) numbers with exponents (this is different from multiply)

Report answers with a reasonable number of significant figures

Unit conversions

Now that we have all the background we need to deal with numbers, it is time to return to units. If we go back to the table we had earlier we see that there are lots of different units in any given category.

Unit category	Metric System	British System
Length	*m, cm, mm*	*mi, in, ft, yds*
Time	*s, hrs, d, ns*	*s, hrs, d, ns*
Volume	*l, dl*	*fl oz, gal, qt*
Mass	*g, kg*	*lbs*

We also know that if we measure something, like your wingspan, that length doesn't depend on the unit system you choose. But the *value* you report does depend on the unit system you choose. My wingspan is 73.2 inches. Or is it 6.10 feet? Or is it 186 centimeters?

Actually it is all of those things. They are all the same. But sometimes we need to convert among different units. Note that you never convert from one unit category to another – you would never measure distance in seconds. But you may convert hours to days.

There are two ways to convert among units. The first involves recognizing the *proportionality* among the units. 1 inch is exactly 2.54 cm. So we could work a problem like 1 inch is to 2.54 cm as 8.3 inches is to *x* cm. I would write that reasoning in a mathematical model or equation as:

$$\frac{1\ inch}{2.54\ cm} = \frac{8.3\ inch}{x\ cm}$$

Notice that the units are the same on both sides – in this case inch divided by cm. We can now use our algebra rules to cross multiply and solve for x:

$$1\ inch \text{ x } x\ cm = 8.3 \text{ inch x } 2.54\ cm$$

$$x\ cm\ =\ \frac{8.3\ inch \text{ x } 2.54\ cm}{1\ inch}\ =\ 21\ cm$$

You can treat units in an algebraic expression the same as you treat variables. So inches/inches is just 1 and you have "eliminated" inches from your expression. Your answer is therefore in cm.

Take a look at the significant figures. We used the *conversion factor* 1 inch = 2.54 cm. When we consider significant figures we do not count the digits of the numbers in conversion factors. Let's think about why. Recall that significant figures are all about *measurement* and the numbers error inherent in the measurement. Conversion factors are assumed to contain no error – they are the exact conversion. They therefore add no error to the reporting – all the error is in the other (the measured) number. Thus you exclude conversion factors from your consideration of significant figures.

The way we converted inches to centimeters is called the proportional reasoning method. This method allows you to really see the proportional relationships in your conversions. However, as your conversion become more complex the proportional method becomes more complex. The proportional method will always work, but let's consider a simpler method using the units (or the dimensions).

<u>Conversions using dimensional analysis</u>

As we begin to bring in mathematics to help us model and solve chemical problems, we will find that "conversions" are very helpful. We won't just convert from one dimension to another, we will also convert from reactants to products – thus the dimensional approach is extremely useful. So let's break it down.

We will always have a *conversion factor.* We are going to write it down. So let's consider the following problem: how many milliliters are there in 12.34 liters of water?

1) We recognize that we have to convert from liters to centimeters – so we need to go find the appropriate conversion factor. (Sometimes this is will be harder than other times). We realize that we are in the same system (metric) and so we look up the metric prefixes and find that:

   1000 ml = 1 l

   We have our conversion factor.

2) We now set up our equation using the dimensions. Start with what we want to know. Remember to include the unit (the dimension) with your variable:

$$x\ milliliters\ = \frac{1000 \text{ m}l}{1\ l}\ x\ 12.34\ liters$$

This can always be done using the following rule:

What you want to know = conversion factor (appropriate units) x what you know

Conversions with multiple conversion factors

Sometimes we cannot find the conversion factor in a table. We can convert from one metric unit to another using a table of metric prefixes. But what if we convert from metric to British? We might need to use more than one conversion factor. Consider this problem:

How many yards are there in 3.45 meters?

Here are my conversion factors. First I find the conversion across systems:

1 inch = 2.54 centimeters

100 centimeters = 1 meter

36 inches = 1 foot

3 feet = 1 yard

Bingo, I have all the conversion factors I need. So I am going to set up the problem as:

What I want to know = conversion factor x conversion factor … x what I know:

$$x\ yards\ = \frac{\text{1 yard}}{3\ ft}\ x\ \frac{\text{1 ft}}{12\ in} x\ \frac{\text{1 in}}{2.54\ cm} x\ \frac{\text{100 cm}}{1\ m} x\ 3.45\ m$$

These can get long and complicated but we don’t have to worry. We lay out the conversion factors based on the dimensions. As long as we keep the numbers and units together of each conversion factor and as long as we make sure our units reduce to 1 (or cancel) we will be fine.

So really now all there is – is practice!